Path integrals for quadratic Lagrangians on real, *p*-adic, and adelic spaces

joint work with B. Dragovich

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Introduction

Formalisms of classical mechanics:

- Hamilton
- Lagrange

Formalisms of quantum mechanics:

- Heisenberg: $$i\hbar\frac{d\hat{A}}{dt} = i\hbar\frac{\partial\hat{A}}{\partial t} + [\hat{A}, \hat{H}],$$
- Schrödinger: $$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = H(\hat{k}, x)\Psi(x,t), \quad \hat{k} = -i\hbar\frac{\partial}{\partial x}.$$
- Feynman:

$$\mathcal{K}(x'', t''; x', t') = \int_{x', t'}^{x'', t''} \exp\left(\frac{2\pi i}{h}\int_{t'}^{t''} L(\dot{q}, q, t)dt\right)\mathcal{D}q,$$

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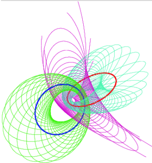

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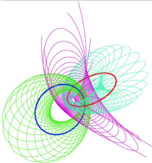

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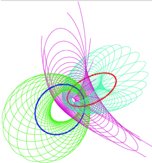

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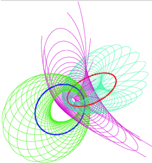

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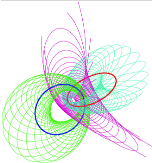

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The probability amplitude $\mathcal{K}(x'', t''; x', t')$ plays cental role in the theory and has the following properties:

$$\int \mathcal{K}(x'', t''; x, t)\mathcal{K}(x, t; x', t')dx = \mathcal{K}(x'', t''; x', t'),$$

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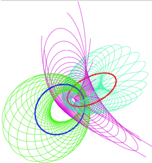

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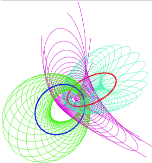

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There is well defined Haar measure and integration, and we have

$$\int_{\mathbb{Q}_p} \chi_p(ayx)\, dx = \delta_p(ay) = |a|_p^{-1}\, \delta_p(y), \quad a \neq 0,$$

$$\int_{\mathbb{Q}_p} \chi_p(\alpha x^2 + \beta x)\, dx = \lambda_p(\alpha)\, |2\alpha|_p^{-\frac{1}{2}}\, \chi_p\left(-\frac{\beta^2}{4\alpha}\right), \quad \alpha \neq 0,$$

where $\delta_p(u)$ is the p-adic Dirac δ function, and for

$x = \sum_{k=m}^{+\infty} x_k p^k, \ x_k = 0, 1, \cdots, p-1, \ x_m \neq 0$, we define $\lambda_p(x)$:

$$\lambda_p(x) = \begin{cases} 1, & m = 2j, \quad p \neq 2, \\ \left(\frac{x_m}{p}\right), & m = 2j+1, \quad p \equiv 1 (\mathrm{mod}\ 4), \\ i\left(\frac{x_m}{p}\right), & m = 2j+1, \quad p \equiv 3 (\mathrm{mod}\ 4), \end{cases}$$

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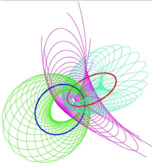

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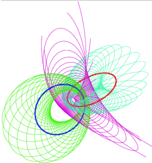

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$$\lambda_2(x) = \begin{cases} \frac{1}{\sqrt{2}}[1 + (-1)^{x_{m+1}} i], & m = 2j, \\ \frac{1}{\sqrt{2}}(-1)^{x_{m+1}+x_{m+2}}[1 + (-1)^{x_{m+1}} i], & m = 2j+1, \end{cases}$$

$\left(\frac{x_m}{p}\right)$ is the Legendre symbol,

$$\left(\frac{a}{p}\right) = \begin{cases} 1, & \text{if} \quad a \equiv y^2 (\text{mod } p), \\ -1, & \text{if} \quad a \not\equiv y^2 (\text{mod } p), \end{cases}$$

$$\lambda_p(a^2 x) = \lambda_p(x), \quad \lambda_p(x)\lambda_p(-x) = 1,$$

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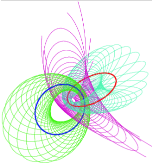
Definition

$$\text{Let} \qquad \Lambda_p(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} \lambda_p(x_i)$$

where subscript $p = \infty, 2, 3, \cdots, p, \cdots$.



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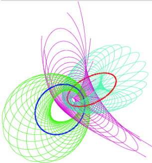

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$$\text{Let} \qquad \Lambda_p(x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} \lambda_p(x_i)$$

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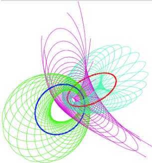

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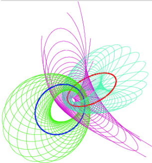

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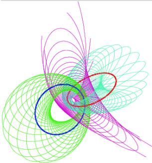

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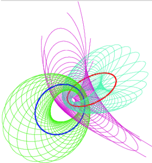

Proposition

The functions $\Lambda_p(x_1\,, x_2\,, \cdots\,, x_n)$ satisfy the properties

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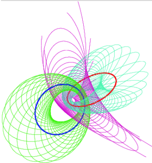

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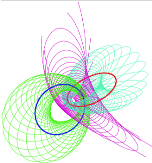

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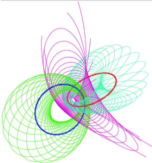

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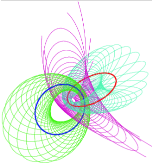

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$$\int_{\mathbb{Q}_v^n}\chi_p(y^TBx)\,d^nx=|\det(B_{kl})|_p^{-1}\prod_{k=1}^{n}\delta_p(y_k)\,,$$

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Proposition

Let $x=(x_1, cdots, x_n),\ \beta=(\beta_1,\cdots,\beta_n)$ be two column vectors, and let $\alpha=(\alpha_{kl})$ be a nonsingular symmetric $n\times n$ matrix, where $x_k,\ \beta_k,\ \alpha_{kl}\in\mathbb{Q}_p$. Then

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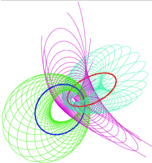

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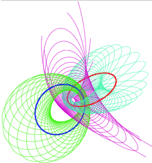

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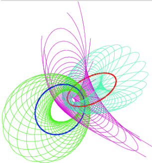

Quadratic Lagrangians

General quadratic Lagrangian has the form

$$L(\dot{q}, q, t) = \frac{1}{2}\,\dot{q}^T A\,\dot{q} + \dot{q}^T B\,q + \frac{1}{2}\,q^T C\,q + D^T\,\dot{q} + E^T\,q_k + \varepsilon$$

Euler-Lagrange equations:

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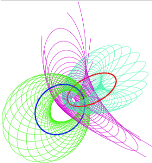
The solution of the above system has the form:

$$q = x(t) = F(t)\,\mathcal{C} + \xi(t),$$

where $F(t) = [f_{km}(t)] \in M_{n,2n}$ is a solutions of the corresponding system of homogeneous differential equations, $\mathcal{C} = [C_m] \in M_{2n,1}$ is the vector of constants, and $\xi(t) = [\xi_k(t)] \in M_{n,1}$ is a particular solution of the system.



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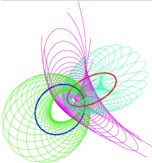

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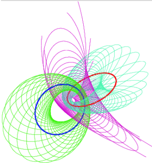

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Quadratic Lagrangians

Lemma

Imposing the boundary conditions $x'_k = x_k(t')$ and $x''_k = x_k(t'')$, the solution of the above system take the form

$$x_k(t) = \frac{1}{\triangle(t'', t')} \sum_{i=1}^{2n} \triangle_i(t'', t') f_{ki}(t) + \xi_k(t)\,, \qquad k = 1, 2, \ldots, n\,.$$

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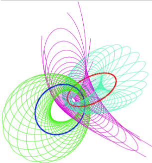
Theorem

Let $\{f_{1j}\ , j = 1, 2, \ldots, 2\,n\}$ be any linearly independent solutions of the resolvent equation for $x_1(t)$, then solutions $f_{km}(t)$ for $x_k(t),\ k \neq 1,$ are determined by the system and the following equality holds

$$\det \begin{bmatrix} F(t) \\ \dot{F}(t) \end{bmatrix} = \frac{\mathcal{D}}{\det A},$$

where $\mathcal{D}$ is an non-zero constant, which could be chosen to be equal to 1.



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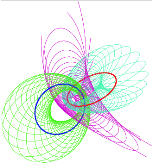
where $\mathcal{D}$ is an non-zero constant, which could be chosen to be equal to 1.



Quadratic Lagrangians

Lemma

Imposing the boundary conditions $x_k' = x_k(t')$ and $x_k'' = x_k(t'')$, the solution of the above system take the form

$$x_k(t) = \frac{1}{\triangle(t'',t')} \sum_{i=1}^{2n} \triangle_i(t'',t') f_{ki}(t) + \xi_k(t)\,, \qquad k = 1, 2, \ldots, n\,.$$

where $\triangle(t'',t') = \det \mathcal{F}$, and $\triangle_i(t'',t')$ has ordinary meaning.

Theorem

Let $\{f_{1j}\ , j = 1, 2, \ldots, 2\,n\}$ be any linearly independent solutions of the resolvent equation for $x_1(t)$, then solutions $f_{km}(t)$ for $x_k(t),\ k \neq 1,$ are determined by the system and the following equality holds

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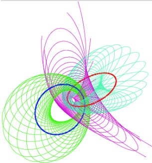

Quadratic Lagrangians and their actions

Zoran Rakić

Theorem

The general form of the action for classical trajectory $x(t)$ of a quadratic Lagrangian, for a particle being in point x' at the time t' and in position x'' at t'', is

$$\bar{S}(x'', t''; x', t') = \frac{1}{2}x''^T\bar{A}x'' + x''^T\bar{B}x' + \frac{1}{2}x'^T\bar{C}x + \bar{D}^Tx'' + \bar{E}^Tx' + \bar{\varepsilon},$$

where $\bar{A} = [\bar{A}_{kl}],\ \bar{B} = [\bar{B}_{kl}],\ \bar{C} = [\bar{C}_{kl}],\ \bar{D} = [\bar{D}_k],$ *and* $\bar{E} = [\bar{E}_k]$

$$\bar{A}_{kl} = \bar{A}_{kl}(t'', t') = \frac{\partial^2\bar{S}_0}{\partial x''_k\partial x''_l}, \quad \bar{B}_{kl} = \bar{B}_{kl}(t'', t') = \frac{\partial^2\bar{S}_0}{\partial x''_k\partial x'_l},$$

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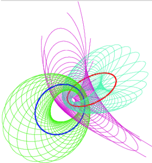
$$\bar{E} = \bar{E}_k(t'', t') = \frac{\partial\bar{S}_0}{\partial x'}, \quad \bar{\varepsilon} = \bar{\varepsilon}(t'', t') = \bar{S}_0.$$

and subscript $_0$ *in the classical action means that after performing derivatives of the* $\bar{S}(x'', t''; x', t')$ *one has to replace* x'' *and* x' *by* $x'' = x' = 0$.



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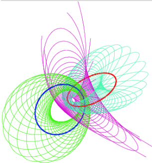

Path Integrals on real and p-adic spaces

The following formula holds

$$\mathcal{K}_p(x'', t''; x', t') = \chi_p\left(-\frac{1}{h}\,\bar{S}(x'', t''; x', t')\right)$$

$$\times \int_{y' \to 0, t'}^{y'' \to 0, t''} \chi_p\left(-\frac{1}{2h}\int_{t'}^{t''}\left(\dot{y}_k\,\frac{\partial}{\partial \dot{q}_k} + y_k\,\frac{\partial}{\partial q_k}\right)^2 L(\dot{q}, q, t)dt\right)\mathcal{D}y,$$

where we used $y'' = y' = 0, \quad S[x] = \bar{S}(x'', t''; x', t')$.

Theorem

(i1) $\mathcal{K}_p(x'', t''; x', t')$ has the form

$$\mathcal{K}_p(x'', t''; x', t') = N_p(t'', t')\,\chi_p\left(-\frac{1}{h}\,\bar{S}(x'', t''; x', t')\right),$$

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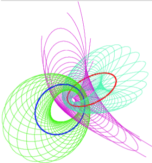
$$\text{(i2)} \quad |N_p(t'', t')|_\infty = \left|\frac{1}{h^n}\det\frac{\partial^2}{\partial x''_k \partial x'_l}\bar{S}_0(x'', t''; x', t')\right|_p^{\frac{1}{2}}.$$



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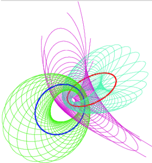

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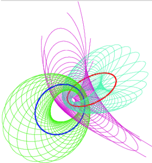

Path integrals on real and p-adic spaces

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where $|\mathcal{A}_p(t'',t')|_\infty = 1$ and $\mathcal{A}_p(t'',t')$ remains to be determined explicitly.

Our expectations.

$$\int \mathcal{K}(x'',t'';x,t)\mathcal{K}(x,t;x',t')dx = \mathcal{K}(x'',t'';x',t'),$$

implies

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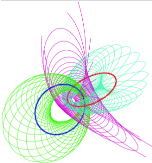

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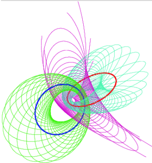

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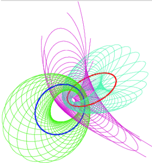

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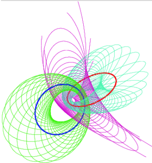

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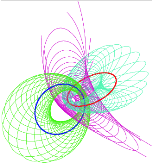

Path integrals on real and p-adic spaces

$$\chi_p\Big(-\frac{1}{h}\big(\frac{1}{2}y''^T A(t'',t')y'' + y''^T B(t'',t')y' + \frac{1}{2}y'^T C(t'',t')y'\big)\Big) \qquad (2)$$

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where

$$z = \frac{y''^T B(t'',t) + y'^T B^T(t,t')}{-h}$$

which imply the third one

$$\mathcal{A}_p(t'',t)\mathcal{A}_p(t,t')\Lambda_p(\alpha_1,\alpha_2,\cdots,\alpha_n) = \mathcal{A}_p(t'',t')\,, \qquad (3)$$

Let us mention here, that for $n = 1$ we have the following solution,

$$\mathcal{A}_p(t'',t') = \lambda_p\left(-\frac{1}{2h}\frac{\partial^2}{\partial y''\partial y'}\bar{S}_0(y'',t'';y',t')\right).$$

Path integrals on real and p-adic spaces

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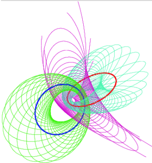

Path integrals on real and p-adic spaces

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Path integrals on real and p-adic spaces

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$$\mathcal{A}_p(t'',t)\mathcal{A}_p(t,t')\Lambda_p(\alpha_1,\alpha_2,\cdots,\alpha_n) = \mathcal{A}_p(t'',t')\,, \tag{3}$$

Let us mention here, that for $n = 1$ we have the following solution,

$$\mathcal{A}_p(t'',t') = \lambda_p\left(-\frac{1}{2h}\,\frac{\partial^2}{\partial y''\partial y'}\,\bar{S}_0(y'',t'';y',t')\right).$$

Expectations

We expect the following solution,

$$\mathcal{A}_p(t'', t') = \lambda_p \left(-\frac{1}{2h} \det \left| \frac{\partial^2}{\partial y'' \partial y'} \bar{S}_0(y'', t''; y', t') \right| \right).$$

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THANK YOU FOR

YOUR ATTENTION !!!